

FAEDO–GALERKIN APPROXIMATE SOLUTIONS FOR NONLOCAL FRACTIONAL DIFFERENTIAL EQUATION OF SOBOLEV TYPE

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Abstract. This paper studies a fractional differential equation of Sobolev type with nonlocal initial conditions in an arbitrary separable Hilbert space. We study the associated integral equation and then, consider a sequence of approximate integral equations obtained by projection of considered associated integral equation onto finite dimensional space. The sufficient condition for providing the existence and uniqueness of mild solution to every approximate integral equation is obtained via the techniques of Banach fixed point theorem and analytic semigroup theory. By utilizing the Faedo-Galerkin approximations, we establish some convergence results for approximate solutions. Finally, an example is given to explain the applicability of the discussed abstract results.

Mathematics subject classification (2010): 26A33, 34K37, 34K40, 34K45, 35R11, 45J05, 45K05.

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