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INFINITELY MANY SOLUTIONS FOR A CLASS OF SUPERQUADRATIC FRACTIONAL HAMILTONIAN SYSTEMS

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Abstract. Applying a variant fountain theorem, we prove the existence of infinitely many solutions for a class of fractional Hamiltonian systems

$$\begin{cases} {}_tD_\infty^\alpha({}_{-\infty}D_t^\alpha u)(t)+L(t)u(t)=\nabla W(t,u(t)),\ t\in\mathbb{R} \\ u\in H^\alpha(\mathbb{R},\ \mathbb{R}^N), \end{cases}$$

where ${}_tD_\infty^\alpha$ and ${}_{-\infty}D_t^\alpha$ are the Liouville-Weyl fractional derivatives of order $\frac{1}{2}<\alpha<1$, $L\in C(\mathbb{R},\mathbb{R}^{N^2})$ is a symmetric matrix-valued function not required to be either uniformly positive definite nor coercive and $W(t,x)\in C^1(\mathbb{R}\times\mathbb{R}^N,\mathbb{R})$ satisfies some weaker superquadratic conditions at infinity in the second variable but does not satisfy the well-known Ambrosetti-Rabinowitz superquadratic growth condition.

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