

APPROXIMATION OF TIME FRACTIONAL BLACK–SCHOLES EQUATION VIA RADIAL KERNELS AND TRANSFORMATIONS

MARJAN UDDIN AND MUHAMMAD TAUFIQ

Abstract. In the present work, a numerical scheme is constructed for approximation of time fractional Black-Scholes model governing European options. The present numerical scheme has the capability to overcome spurious oscillation in the case of volatility. In the present numerical method, the Laplace transform, radial kernels and quadrature rule are used. The time variable is eliminated by the use of Laplace transform which significantly reduced the computational cost as compared to the time-marching schemes. The spatial operator is discretized using radial kernels in the local setting which results in sparse differentiation matrices. By Laplace transform the solution is represented as integral along a smooth contour in the complex plane which is then evaluated by quadrature. The proposed numerical scheme is used to price several different European options.

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