

AN ORDERING ON GREEN'S FUNCTION AND A LYAPUNOV-TYPE INEQUALITY FOR A FAMILY OF NABLA FRACTIONAL BOUNDARY VALUE PROBLEMS

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Abstract. In this article, we consider a family of two-point Riemann–Liouville type nabla fractional boundary value problems involving a fractional difference boundary condition. We construct the corresponding Green's function and deduce its ordering property. Then, we obtain a Lyapunov-type inequality using the properties of the Green's function, and illustrate a few of its applications.

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