

SOME PERTURBED VERSIONS OF THE GENERALIZED TRAPEZOID TYPE INEQUALITIES FOR TWICE DIFFERENTIABLE FUNCTIONS

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Abstract. In this study, we first obtain an identity for twice differentiable functions. Then we establish some new perturbed trapezoid type integral inequalities for functions whose first derivatives either are of bounded variation or Lipschitzian. Moreover, some perturbed versions of trapezoid type inequalities for mapping whose second derivatives are bounded, of bounded variation or Lipschitzian, respectively.

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