

ON AN INTEGRO–DIFFERENTIAL EQUATION OF ARBITRARY (FRACTIONAL) ORDERS WITH NONLOCAL INTEGRAL AND INFINITE–POINT BOUNDARY CONDITIONS

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Abstract. In this paper, we study the existence and uniqueness of solutions for an integro-differential equation of arbitrary (fractional) orders with nonlocal integral and infinite-point boundary conditions, continuous dependence of the solution on nonlocal data, on initial condition and on functional equation also will be study. An examples to prove main results.

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