

## ESTIMATES INVOLVING THE $\omega$ -RIEMANN—LIOUVILLE FRACTIONAL INTEGRAL OPERATORS BY MEANS OF $\eta$ -QUASICONVEXITY WITH APPLICATIONS TO MEANS

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*Abstract.* Since not every quasiconvex function is convex, it is our purpose in this present paper to extend some already established inequalities of the Hermite–Hadamard–Fejér type and its companions for convex functions to the class of  $\eta$ -quasiconvex functions. The new results obtained herein are in terms of the  $\omega$ -Riemann–Liouville fractional integral operators and they reduce to inequalities for quasiconvex functions for a particular choice of the bifunction  $\eta$ . In addition, we apply some of our results to certain special means of positive real numbers to obtain more estimates in this regard.

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