

THE GENERALIZED ABEL'S INTEGRAL EQUATIONS ON R^n WITH VARIABLE COEFFICIENTS

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Abstract. The convergent and stable solutions are constructed in the space of Lebesgue integrable functions for the generalized Abel's integral equations of the second kind with variable coefficients on R^n . Several applicable examples are presented, including one solving the fractional partial differential equation with the initial condition.

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