

## WEAK SOLUTIONS FOR CAPUTO-PETTIS FRACTIONAL $q$ -DIFFERENCE INCLUSIONS

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*Abstract.* This article deals with some existence of weak solutions for a class of Caputo fractional  $q$ -difference inclusions and a coupled system of Caputo fractional  $q$ -difference inclusions by using the set-valued analysis, and Mönch's fixed point theorem associated with the technique of measure of weak noncompactness. Two illustrative examples are given in the end.

*Mathematics subject classification* (2010): 26A33.

*Keywords and phrases:* Pettis  $q$ -difference inclusion, Caputo fractional derivative, measure of weak noncompactness, coupled system, weak solution, fixed point.

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