

## ON THE STABILITY AND STABILIZATION OF SOME SEMILINEAR FRACTIONAL DIFFERENTIAL EQUATIONS IN BANACH SPACES

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*Abstract.* In this paper, we prove the existence and uniqueness of a mild solution to a class of semilinear fractional differential equation in an infinite Banach space with Caputo derivative order  $0 < \alpha \leq 1$ . Furthermore, we establish the stability conditions and then prove that the considered initial value problem is exponentially stabilizable when the stabilizer acts linearly on the control system.

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