

## OSTROWSKI AND TRAPEZOID TYPE INEQUALITIES FOR RIEMANN–LIOUVILLE FRACTIONAL INTEGRALS OF ABSOLUTELY CONTINUOUS FUNCTIONS WITH BOUNDED DERIVATIVES

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*Abstract.* In this paper we establish some Ostrowski and trapezoid type inequalities for the Riemann–Liouville fractional integrals of absolutely continuous functions with bounded derivatives. Applications for mid-point and trapezoid inequalities are provided as well. They generalize the known results holding for the classical Riemann integral. Some examples for convex functions are also given.

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