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SOLUTION OF TIME–SPACE FRACTIONAL BLACK–SCHOLES EUROPEAN OPTION PRICING PROBLEM THROUGH FRACTIONAL REDUCED DIFFERENTIAL TRANSFORM METHOD

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Abstract. Mathematical model introduced by Black and Scholes express financial derivatives more significantly. This model with fractional derivatives resulting in fractional Black-Scholes (B-S) equation express financial problems in a better way. In this paper, we introduce the fractional reduced differential transform method (FRDTM) to solve the time-space fractional Black-Scholes equation executing European options. This method is a modified version of the original differential transform method (DTM). This method proves to be valid for solving time-space Black-Scholes equation as it reduces the computational work to a greater extent. Moreover, this method helps in finding the solution without linearization or discretization. The efficiency of the method is tested by solving certain examples. The proposed mathematical representation can be useful to understand and solve time-space fractional differential equations arising in financial mathematics and other related fields.

Mathematics subject classification (2020): 91G20, 91G30, 91G80, 35Q91.
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