

## EXISTENCE RESULTS FOR A CAPUTO–HADAMARD TYPE FRACTIONAL BOUNDARY VALUE PROBLEM

ABDELOUAHEB ARDJOUNI

*Abstract.* The main purpose of this paper is to establish the existence and uniqueness of mild solutions for a Caputo–Hadamard type fractional boundary value problem. Existence and uniqueness results are based on the Krasnoselskii fixed point theorem and the Banach contraction mapping principle. Finally, two examples are given to illustrate this work.

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