

GENERALIZED FRACTIONAL OSTROWSKI TYPE INEQUALITIES VIA $(\alpha, \beta, \gamma, \delta)$ -CONVEX FUNCTIONS

ALI HASSAN* AND ASIF RAZA KHAN

Abstract. We are introducing very first time a generalized class named it the class of $(\alpha, \beta, \gamma, \delta)$ -convex functions in mixed kind. This generalized class contains many subclasses including class of (α, β) -convex functions in 1st and 2nd kind, (s, r) -convex functions in mixed kind, s -convex functions in 1st and 2nd kind, P -convex function, quasi convex functions and the class of ordinary convex. Also, we would like to state the generalization of the classical Ostrowski inequality via fractional integrals with respect to another function, which is obtained for functions whose first derivative in absolute values is $(\alpha, \beta, \gamma, \delta)$ -convex functions in mixed kind. Moreover we establish some Ostrowski type inequalities via fractional integrals with respect to another function and their particular cases for the class of functions whose derivatives in absolute values at certain powers are $(\alpha, \beta, \gamma, \delta)$ -convex functions in mixed kind by using different techniques including Hölder's inequality and power mean inequality. Also, standard results would be capture as special cases. Moreover, some applications in terms of special means would also be given.

Mathematics subject classification (2020): 26A33, 26A51, 26D15, 26D99, 47A30, 33B10.

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