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ON SOLVABILITY OF THE NON–LOCAL PROBLEM FOR THE FRACTIONAL MIXED–TYPE EQUATION WITH BESSEL OPERATOR

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Abstract. The m-point non-local problem is considered for the partial differential equation of mixed-type with singular coefficients, namely fractional wave equation involving the right-hand side bi-ordinal Hilfer derivative and sub-diffusion equation with the regularized Caputo-like counterpart hyper-Bessel differential operator. The main technique of solving the problem is based on the method of separation variables. Also, the connection between the given data and the uniquely solvability of the problem is established and an explicit solution is represented by Fourier-Bessel series in the given domain.

Mathematics subject classification (2020): 35M12, 35R11.
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