

FRACTIONAL CALCULUS AND FAMILIES OF GENERALIZED LEGENDRE–LAGUERRE–APPELL POLYNOMIALS

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Abstract. In this article, new families of the generalized Legendre-Laguerre-Appell polynomials are introduced using a combination of operational definitions and integral representations. The integral transformations and the appropriate operational rules are used to obtain the explicit summation equations, determinant definitions, and recurrence relations for the generalised Legendre-Laguerre-Appell polynomials. For the generalized Legendre-Laguerre-Bernoulli, Legendre-Laguerre-Euler, and Legendre-Laguerre-Genocchi polynomials, an equivalent investigation of these findings is offered. Additionally, a number of identities for these polynomials are derived by using suitable operational definitions.

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