

HÖLDER AND MINKOWSKI TYPE INEQUALITIES FOR PSEUDO-FRACTIONAL INTEGRAL

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Abstract. We first introduce the concept of fractional operators and pseudo-analysis. Then we present new versions of Hölder, Minkowski and reverse Minkowski inequalities via ψ -Riemann-Liouville-Mittag-Leffler pseudo-fractional integral on a semiring $([a, b], \oplus, \odot)$.

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