

ANALYTICAL AND NUMERICAL STUDY OF A LINEAR COUPLED SYSTEM INVOLVING CAPUTO–FABRIZIO FRACTIONAL DERIVATIVE WITH BOUNDARY CONDITIONS

IKRAM MANSOURI, MOHAMMED MOUMEN BEKKOUCHE,
ABDELAZIZ AZEB AHMED, FARES YAZID*
AND FATIMA SIHAM DJERADI

Abstract. This article is concerned with a coupled system of linear fractional differential equations of Caputo-Fabrizio type conformable fractional derivation with boundary conditions. In order to prove the existence and uniqueness of the solution, the problem is transformed into an equivalent linear Volterra-Fredholm integral equations of the second kind, and by using the Banach's fixed point theory the existence and uniqueness of solutions are obtained. Finally, the analytical results are supported by numerical results to illustrate the obtained results.

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