

FRACTIONAL NON-AUTONOMOUS EVOLUTION EQUATIONS WITH INTEGRAL IMPULSE CONDITION IN FRÉCHET SPACES

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Abstract. The primary focus of this study is to explore the presence of a mild solution within a specific category of fractional non-autonomous differential evolution equations, incorporating integral impulse conditions. The approach employed extends the classical Darbo fixed point theorem for Fréchet spaces, leveraging the notion of a measure of noncompactness along with the principle of K -set contraction. To illustrate our findings, we offer an illustrative example.

Mathematics subject classification (2020): 26A33, 47H10, 34K37.

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