

ON GENERALIZED CAPUTO'S FRACTIONAL ORDER FUZZY ANTI PERIODIC BOUNDARY VALUE PROBLEM

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Abstract. In this article, we consider an anti periodic fuzzy boundary value problem with order α , where $1 < \alpha < 2$, under newly defined generalized Caputo's fractional derivative (called as OBC) and study the existence and uniqueness of the solution for the considered problem via fixed point technique. Also, we illustrate the results with some examples involving the developed numerical technique based on fractional Euler's method of integration.

Mathematics subject classification (2020): 26A33, 30E25, 03E72, 34A07, 47H10.

Keywords and phrases: Fractional derivatives and integrals, boundary value problems, theory of fuzzy sets, fuzzy ordinary differential equations, fixed-point theorems.

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