Journal of
Classical
Analysis
Volume 1, Number 1 (2012), 1–16
doi:10.7153/jca-01-01

THE ASYMPTOTICS OF THE MITTAG–LEFFLER POLYNOMIALS

R. B. PARIS

Abstract. We investigate the asymptotic behaviour of the Mittag-Leffler polynomials $G_n(z)$ for large n and z, where z is a complex variable satisfying $0 \leqslant \arg z \leqslant \frac{1}{2}\pi$. A summary of the asymptotic properties of $G_n(ix)$ for real values of x and an approximation for its extreme zeros as $n \to \infty$ are given. When the variables are such that z/n is finite, an expansion is obtained using the method of steepest descents applied to a suitable integral representation. This expansion holds everywhere in the first quadrant of the z-plane except in the neighbourhood of the point $z = in$, where there is a coalescence of saddle points. Numerical results are presented to illustrate the accuracy of the various expansions.

Mathematics subject classification (2010): Primary 41A60, 33C45.

Keywords and phrases: Mittag-Leffler polynomials, asymptotic expansion, uniform approximation, extreme zeros.

REFERENCES

[1] H. BATEMAN, *The polynomial of Mittag-Leffler*, Proc. N. A. S. **26** (1940), 491–496.

[2] H. BATEMAN, *An orthogonal property of the hypergeometric polynomial*, Proc. N. A. S. **28** (1942), 374–377.

[3] N. BLEISTEIN AND R. A. HANDELSMAN, *Asymptotic Expansion of Integrals*, Dover, New York, 1986.

[4] R. B. DINGLE, *Asymptotic Expansions: Their Derivation and Interpretation*, Academic Press, London, 1973.

[5] T. X. HE, L. C. HSU AND P. J.-S. SHIUE, *The Scheffer group and the Riordan group*, Discrete Appl. Math. **155** (2007), 1895–1909.

[6] X. LI AND R. WONG, *On the asymptotics of the Meixner-Pollaczek polynomials and their zeros*, Constr. Approx. **17** (2001), 59–90.

[7] Y. L. LUKE, *The Special Functions and Their Approximations*, Vol. 1, Academic Press, New York, 1969.

[8] F. W. J. OLVER, *Asymptotics and Special Functions*, Academic Press, New York, 1974. Reprinted in A. K. Peters, Massachussets, 1997.

[9] F. W. J. OLVER, D. W. LOZIER, R. F. BOISVERT AND C. W. CLARK (EDS.), *NIST Handbook of Mathematical Functions*, Cambridge University Press, Cambridge, 2010.

[10] R. B. PARIS, *Hadamard Expansions and Hyperasymptotic Evaluation: an Extension of the Method of Steepest Descents*, Encyclopedia of Mathematics, vol. 141, Cambridge University Press, Cambridge, 2011.

[11] S. ROMAN, *The Umbral Calculus*, Academic Press, New York, 1984.

[12] G. RZĄDKOWSKI, *On some expansions, involving falling factorials, for the Euler gamma function and the Riemann zeta function*, arXiv.org/abs/1007.1955v1 (2010).

[13] M. S. STANKOVIĆ, S. D. MARINKOVIĆ AND P. M. RAJKOVIĆ, *Deformed Mittag-Leffler polynomials*, arXiv.org/abs/1007.3612v1 (2010).

[14] G. N. WATSON, *Asymptotic expansions of hypergeometric functions*, Trans. Cambridge Phil. Soc. **22** (1918), 277–308.

[15] E. T. WHITTAKER AND G. N. WATSON, *Modern Analysis*, Cambridge University Press, Cambridge 1952.

[16] R. WONG, *Asymptotic Expansion of Integrals*, Academic Press, London, 1989.