

ON THE ASYMPTOTIC EXPANSION OF A BINOMIAL SUM INVOLVING POWERS OF THE SUMMATION INDEX

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Abstract. Work elsewhere [1, 3] has revealed the leading asymptotic behaviour of the binomial sum $S_p(n)$ defined by

$$S_p(n) = \sum_{j=1}^n j^p \binom{n+j}{j}$$

in the limit $n \rightarrow \infty$ in the case of positive integer p . In this paper, we establish the asymptotic expansion of $S_p(n)$ first for positive integer p and secondly, by means of an integral representation for the sum, for arbitrary values of the index p .

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