

ON THE UPPER BOUND OF THE NUMBER OF REAL ZEROS OF A RANDOM ALGEBRAIC POLYNOMIAL

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Abstract. Let N_n be the number of level crossings of a random algebraic curve $f(x, w) = \sum_{r=0}^n a_r \xi_r(w) x^r$ where the co-efficients $\xi_r(w)$'s are identically distributed independent random variables following semi-stable distribution with characteristic function $\exp(-C + \cos \log |t|)|t|^\alpha$ for $0 < \alpha \leq 2$ and $C > 1$. It is proved that $N_n \leq \mu (\log n)^2$ in the weak version outside a set of measure less than $\frac{\mu^*}{n^{3\alpha-1-\varepsilon}} + \frac{\mu^{**}}{n^{1-\varepsilon}}$ where $0 < \varepsilon < 1$, and $N_n \leq \mu (\log n)^3$ in the strong version according to the sense of Evans, outside a set of measure less than $\frac{\mu'}{n_0^{\alpha \log n_0 - 2 - \varepsilon}} + \frac{\mu''}{n_0^{\log n_0 - 1 - \varepsilon}}$ where $0 < \alpha \leq 2$ and $0 < \varepsilon < 1$ for all $n \geq n_0$.

Mathematics subject classification (2010): 60AXX.

Keywords and phrases: Random variables, Random algebraic equations; Semi-stable distribution; Real roots.

REFERENCES

- [1] EVANS, E. A., *On the number of real roots of a random algebraic equation*, Proc. London Math. Soc., 15 (1965) 731–749.
- [2] GNEDENKO, B. V. AND KOLMOGOROV, A. N., *Limit distributions for sums of independent random variables*, Addison-Wesley Publishing Company, Inc., Cambridge, Mass., 1954.
- [3] LOEVE, M., *Probability Theory*, 1963.
- [4] LOGAN, B. F. AND SHEPP, L. A., *Real zeros of random polynomials*, Proc. London Math. Soc., **18**, 3 (1968) 29–35.
- [5] NAYAK, N. N. AND DAS, B. K., *Real zeros of a random algebraic equation*, Tamkang J. Math., **17**, 1 (1986) 75–86.
- [6] NAYAK, N. N., MISHRA AND MOHANTY, S. P., *On the lower bound of the number of real zeros of a random algebraic polynomial*, J. Indian Math. Soc. (N.S.) **49** (1985), 1-2, 7–15 (1987).
- [7] SAMAL, G. AND MISHRA M. N., *On the upper bound of the number of real roots of a random algebraic equation with infinite variance*, J. London Math. Soc. **6**, 2 (1973) 598–604.
- [8] SAMBANDHAM, M., *Real zeros of a random polynomial with hyperbolic elements*, Indian J. Pure Appl. Math. **7**, 5 (1976) 553–556.
- [9] SHIMIZU, R., *Certain class of infinitely divisible characteristic functions*, Ann. Inst. Statist. Math. **17** (1965) 115–132.
- [10] SHIMIZU, R., *Characteristic functions satisfying a functional equation. I*, Ann. Inst. Statist. Math. **20** (1968) 187–209.
- [11] SHIMIZU, R., *Characteristic functions satisfying a functional equation. II*, Ann. Inst. Statist. Math. **21** (1969) 391–405.
- [12] SHIMIZU, RYOICHI, *On the domain of partial attraction of semi-stable distributions*, Ann. Inst. Statist. Math. **22** (1970), 245–255.