

## FOURIER EXPANSIONS FOR A LOGARITHMIC FUNDAMENTAL SOLUTION OF THE POLYHARMONIC EQUATION

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*Abstract.* In even-dimensional Euclidean space for integer powers of the Laplacian greater than or equal to the dimension divided by two, a fundamental solution for the polyharmonic equation has logarithmic behavior. We give two approaches for developing an azimuthal Fourier expansion of this logarithmic fundamental solution. The first approach is algebraic and relies upon the construction of two-parameter polynomials which we call logarithmic polynomials. The second approach depends on the computation of parameter derivatives of Fourier series expressions for a power-law fundamental solution of the polyharmonic equation. We conclude by comparing the two approaches and giving the azimuthal Fourier series for a logarithmic fundamental solution of the polyharmonic equation in rotationally-invariant coordinate systems.

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