

## GENERALIZATIONS AND SPECIALIZATIONS OF GENERATING FUNCTIONS FOR JACOBI, GEGENBAUER, CHEBYSHEV AND LEGENDRE POLYNOMIALS WITH DEFINITE INTEGRALS

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*Abstract.* In this paper we generalize and specialize generating functions for classical orthogonal polynomials, namely Jacobi, Gegenbauer, Chebyshev and Legendre polynomials. We derive a generalization of the generating function for Gegenbauer polynomials through extension a two element sequence of generating functions for Jacobi polynomials. Specializations of generating functions are accomplished through the re-expression of Gauss hypergeometric functions in terms of less general functions. Definite integrals which correspond to the presented orthogonal polynomial series expansions are also given.

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