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ON SOME FURTHER HYPERGEOMETRIC SERIES IDENTITIES OBTAINED VIA FRACTIONAL CALCULUS

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Abstract. In this paper we present a generalization of a result obtained recently by Rathie and Kiliçman (A. K. Rathie and A. Kiliçman, On certain new hypergeometric identities, Preprint 2014) involving hypergeometric identities. The result is obtained by suitably applying fractional calculus technique to a generalization of a quadratic transformation for the Gauss hypergeometric function due to Gauss.

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