

REVIVING THE QUADRATIC SERIES OF AU-YEUNG

CORNEL IOAN VĂLEAN AND OVIDIU FURDUI

Abstract. In this paper we revive and bring to light the quadratic series of Au–Yeung

$$\sum_{n=1}^{\infty} \left(\frac{H_n}{n} \right)^2 = \frac{17}{4} \zeta(4),$$

where H_n denotes the n th harmonic number. We prove this series identity by using a technique based on the computation of a special logarithmic integral combined with Abel's summation formula.

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