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ASYMPTOTIC EXPANSIONS PERTAINING TO THE LOGARITHMIC SERIES AND RELATED TRIGONOMETRIC SUMS

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Abstract. The partial sum of the Maclaurin series of $-\ln(1-z)$ is $f_n(z) \equiv \sum_{k=1}^{n-1} z^k/k$. We find concise closed-form expressions, involving Eulerian polynomials, for the full asymptotic expansion of $f_n(z)$ as $n \to \infty$. We then use our expressions to find large-n compound asymptotic expansions, involving real quantities only, for $c_n(\theta) \equiv \sum_{k=1}^{n-1} \cos k\theta/k$, $s_n(\theta) \equiv \sum_{k=1}^{n-1} \sin k\theta/k$, $r_n(\theta) \equiv \sum_{k=0}^{n-1} (-1)^k \cos[(2k+1)\theta]/(2k+1)$, and a number of other trigonometric sums. Many of these sums are ubiquitous in the literature on the Gibbs phenomenon in the context of Fourier series.

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