

LEGENDRE–TYPE RELATIONS FOR GENERALIZED COMPLETE ELLIPTIC INTEGRALS

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Abstract. Legendre’s relation for the complete elliptic integrals of the first and second kinds is generalized. The proof depends on an application of the generalized trigonometric functions and is alternative to the proof for Elliott’s identity.

Mathematics subject classification (2010): 33E05, 33C75, 34L10.

Keywords and phrases: Legendre’s relation, complete elliptic integrals, generalized trigonometric functions, Elliott’s identity.

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