

CLOSURE OF THE LINEAR SPAN OF AN EXPONENTIAL SYSTEM IN A WEIGHTED BANACH SPACE

ELIAS ZIKKOS

Abstract. For a certain class of sequences with repeated terms,

$$\{\lambda_n, \mu_n\}_{n=1}^{\infty} := \underbrace{\{\lambda_1, \lambda_1, \dots, \lambda_1\}}_{\mu_1 \text{ times}}, \underbrace{\{\lambda_2, \lambda_2, \dots, \lambda_2\}}_{\mu_2 \text{ times}}, \dots, \underbrace{\{\lambda_k, \lambda_k, \dots, \lambda_k\}}_{\mu_k \text{ times}}, \dots,$$

we prove that every function belonging to the closed span of the exponential system

$$\{x^k e^{\lambda_n x} : n \in \mathbb{N}, k = 0, 1, 2, \dots, \mu_n - 1\},$$

in some weighted Banach spaces on the real line, extends analytically as an entire function by admitting a series representation of the form

$$\sum_{n=1}^{\infty} \left(\sum_{k=0}^{\mu_n-1} c_{n,k} z^k \right) e^{\lambda_n z}, \quad c_{n,k} \in \mathbb{C}, \quad \forall z \in \mathbb{C}.$$

Mathematics subject classification (2010): 30B50, 30B60, 46E15, 46E20.

Keywords and phrases: Completeness, closure, minimality, Taylor-Dirichlet series.

REFERENCES

- [1] J. M. ANDERSON, K. G. BINMORE, *Closure theorems with applications to entire functions with gaps*, Trans. Amer. Math. Soc. **161** (1971) 381–400.
- [2] G. T. DENG, *Incompleteness and closure of a linear span of exponential system in a weighted Banach space*, J. Approx. Theory **125** no. 1 (2003), 1–9.
- [3] W. H. J. FUCHS, *On the closure of $\{e^{-t} t^{an}\}$* , Proc. Cambridge Philos. Soc. **42** (1946), 91–105.
- [4] P. MALLIAVIN, *Sur quelques procédés d'extrapolation*, Acta Math. **93** (1955) 179–255.
- [5] J. NING, G. T. DENG, C. YI, *Incompleteness and closure of the multiplicity system $\{t^k e^{\lambda_j t}\}$ in the weighted Banach space*, J. Math. Anal. Appl. **341** no. 2 (2008), 1007–1017.
- [6] X. YANG, *Incompleteness of exponential system in the weighted Banach space*, J. Approx. Theory **153** no. 1 (2008), 73–79.
- [7] E. ZIKKOS, *Completeness of an exponential system in weighted Banach spaces and closure of its linear span*, J. Approx. Theory **146** no. 1 (2007), 115–148.