

ANALYTICAL, ASYMPTOTIC AND INTEGRAL REPRESENTATIONS FOR A DOUBLE SUM

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Abstract. We give an analytical representation for the double sum

$$\sum_{1 \leq i < j \leq n} \frac{1}{[(n+i)(n+j)]^k}$$

in terms of the polygamma functions, where k is any given positive integer. Based on this result, we present an asymptotic formula as $n \rightarrow \infty$ and an integral representation for this sum.

Mathematics subject classification (2010): Primary 41A60; Secondary 40A05.

Keywords and phrases: Psi function; Polygamma functions; Bernoulli numbers; Asymptotic formula.

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