

ON THE CONVERGENCE OF SERIES WITH RECURSIVELY DEFINED TERMS

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Abstract. We investigate the asymptotic behavior of a sequence $(x_n)_{n=0}^{\infty}$ defined recursively by $x_{n+1} = f(x_n)$, $n \geq 0$ where $f: [0, \infty) \rightarrow [0, \infty)$ is a continuous function. A fundamental criterion on the function $f(x)$ for estimating the rate of decay of x_n as n tends to ∞ and for testing convergence of the series $\sum_{n=0}^{\infty} x_n$ is proposed and justified. Criteria for testing absolute and conditional convergence of $\sum_{n=0}^{\infty} x_n$ when $f(x)$ is not a non-negative function are also formulated and proved.

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