

## ON A SUBCLASS OF CLOSE-TO-CONVEX HARMONIC MAPPINGS

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**Abstract.** For  $\alpha > -1$  and  $\beta > 0$ , let  $\mathcal{B}_{\mathcal{H}}^{\alpha, \beta}$  denote the class of sense preserving harmonic mappings  $f = h + \bar{g}$  in the open unit disk  $\mathbb{D}$  satisfying  $|zh''(z) + \alpha(h'(z) - 1)| \leq \beta - |zg''(z) + \alpha g'(z)|$ . First, we establish that each function belonging to this class is close-to-convex in the open unit disk if  $\beta \in (0, 1 + \alpha]$ . Next, we obtain coefficient bounds, growth estimates and convolution properties. We end the paper with applications and will construct harmonic univalent polynomials belonging to this class.

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