

ON THE GENERALIZED HURWITZ–LERCH ZETA FUNCTION AND GENERALIZED LAMBERT TRANSFORM

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Abstract. Raina and Srivastava [20] introduced a generalized Lambert transform. Goyal and Laddha [8] have introduced generalizations of the Riemann zeta function and generalized Lambert transform. In the present paper, we introduce generalizations of the Hurwitz–Lerch zeta function and Lambert transform in a diverse direction. We derive generating functions involving generalized Hurwitz–Lerch zeta function. Connections between the generalized Lambert transform and generalized Hurwitz–Lerch zeta function are established. An inversion formula for the generalized Lambert transform is obtained. Some examples and special cases to illustrate our results are also mentioned.

Mathematics subject classification (2010): 11M06, 11M35, 33C10.

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