

## SIMULTANEOUS APPROXIMATION PROPERTIES OF DE LA VALLÉE-POUSSIN MEANS IN WEIGHTED ORLICZ SPACES

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*Abstract.* We investigate the simultaneous approximation properties of the de la Vallée-Poussin means in weighted Orlicz spaces in terms of the modulus of smoothness. In terms of the modulus of smoothness the direct theorem of simultaneous approximation is proved. Also, in weighted Orlicz spaces the modulus of smoothness are estimated from below and above in terms of  $n$ -th partial Fourier sums and de la Vallée-Poussin means.

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