

UNIFORM NORM ESTIMATES OF BERNSTEIN-TYPE FOR LACUNARY-TYPE COMPLEX POLYNOMIALS

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Abstract. In this paper, we prove some inequalities for the uniform norm of the derivative and polar derivative of lacunary-type complex polynomials having zeros in closed exterior of a circle of arbitrary positive radius. The results obtained besides extend some classical Bernstein-type inequalities also include several interesting generalizations and refinements of some known inequalities involving the polar derivative.

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