ALTERNATING EULER SUMS AND BBP–TYPE SERIES

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Abstract. An investigation into a family of definite integrals containing log-polylog functions with negative argument will be undertaken in this paper. It will be shown that Euler sums play an important part in the solution of these integrals and some may be represented as a BBP type formula. In a special case we prove that the corresponding log integral can be represented as a linear combination of the product of zeta functions and the Dirichlet beta function.

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