

IMPROVED UPPER BOUND OF THIRD ORDER HANKEL DETERMINANT FOR OZAKI CLOSE-TO-CONVEX FUNCTIONS

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Abstract. In this paper we improve the upper bound of the third order Hankel determinant for the class of Ozaki close-to-convex functions. The sharp bound is conjectured.

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