

PROJECTIONS OF MUTUAL MULTIFRACTAL FUNCTIONS

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Abstract. The aim of this article is to study the behavior of the relative multifractal spectrum under projections. First of all, we depict a relationship between the mutual multifractal spectra of a couple of measures (μ, ν) and its orthogonal projections in Euclidean space. As an application, we improve Svetova's results in [46] and study the mutual multifractal analysis of the projections of measures.

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REFERENCES

- [1] M. ABID, M. SLIMANE, I. OMRANE AND B. HALOUANI, *Mixed wavelet leaders multifractal formalism in a product of critical besov spaces*, *Mediterr. J. Math.*, **14**, (2017), 1–20.
- [2] N. ATTIA AND B. SELMI, *Relative multifractal box-dimensions*, *Filomat*, **33**, (2019), 2841–2859.
- [3] N. ATTIA, B. SELMI AND CH. SOUSSI, *Some density results of relative multifractal analysis*, *Chaos, Solitons and Fractals*, **103**, (2017), 1–11.
- [4] L. BARREIRA AND B. SAUSSOL, *Variational principles and mixed multifractal spectra*, *Trans. Amer. Math. Soc.*, **353**, (2001) 3919–3944.
- [5] L. BARREIRA AND P. DOUTOR, *Birkhoff Averages for Hyperbolic Flows: Variational Principles and Applications*, *Journal of Statistical Physics*, **115**, (2004), 1567–1603.
- [6] J. BARRAL AND I. BHOURI, *Multifractal analysis for projections of Gibbs and related measures*, *Ergodic Theory and Dynamic systems*, **31**, (2011), 673–701.
- [7] J. BARRAL AND D. J. FENG, *Projections of planar Mandelbrot random measures*, *Advances in Mathematics*, **325**, (2018), 640–718.
- [8] F. BEN NASR, I. BHOURI AND Y. HEURTEAUX, *The validity of the multifractal formalism: results and examples*, *Adv. in Math.*, **165**, (2002), 264–284.
- [9] J. COLE, *Relative multifractal analysis*, *Chaos, Solitons and Fractals*, **11**, (2000), 2233–2250.
- [10] C. DAI AND Y. LI, *A multifractal formalism in a probability space*, *Chaos Solitons Fractals*, **27**, (2006), 57–73.
- [11] M. DAI, *Mixed self-conformal multifractal measures*, *Analysis in Theory and Applications*, **25**, (2009), 154–165.
- [12] M. DAI AND Y. SHI, *Typical behavior of mixed L^q -dimensions*, *Nonlinear Analysis: Theory, Methods & Applications*, **72**, (2010), 2318–2325.
- [13] M. DAI AND W. LI, *The mixed L^q -spectra of self-conformal measures satisfying the weak separation condition*, *J. Math. Anal. Appl.*, **382**, (2011), 140–147.
- [14] M. DAI, C. WANG AND H. SUN, *Mixed generalized dimensions of random self-similar measures*, *Int. J. Nonlinear. Sci.*, **13**, (2012), 123–128.
- [15] Z. DOUZI AND B. SELMI, *Multifractal variation for projections of measures*, *Chaos, Solitons and Fractals*, **91**, (2016), 414–420.
- [16] Z. DOUZI AND B. SELMI, *On the projections of the mutual multifractal Rényi dimensions*, *Anal. Theory Appl.*, **37**, (2021), 572–592.
- [17] Z. DOUZI AND B. SELMI, *A relative multifractal analysis: box-dimensions, densities, and projections*, *Quaestiones Mathematicae*, (to appear), <http://doi.org/10.2989/16073606.2021.1941375>.
- [18] K. J. FALCONER, *The Geometry of Fractal sets*, Cambridge univ. Press New. York-London., (1985).

- [19] K. J. FALCONER AND J. D. HOWROYD, *Packing Dimensions of Projections and Dimensions Profiles*, Math. Proc. Cambridge Philos. Soc., **121**, (1997), 269–286.
- [20] K. J. FALCONER AND P. MATTILA, *The Packing Dimensions of Projections and Sections of Measures*, Math. Proc. Cambridge Philos. Soc., **119**, (1996), 695–713.
- [21] M. HOCHMAN AND P. SHMERKIN, *Local entropy averages and projections of fractal measures*, Annals Math., **175**, (2012), 1001–1059.
- [22] R. KAUFMAN, *On Hausdorff dimension of projections*, Mathematika, **15**, (1968), 153–155.
- [23] M. KHELIFI, H. LOTFI, A. SAMTI AND B. SELMI, *A joint multifractal analysis*, Chaos, Solitons & Fractals, **140**, (2020), 110091.
- [24] Z. LI AND B. SELMI, *On the multifractal analysis of measures in a probability space*, Illinois J. Math., **65**, (2021), 687–718.
- [25] J. M. MARSTRAND, *Some fundamental geometrical properties of plane sets of fractional dimensions*, Proceedings of the London Mathematical Society, **4**, (1954), 257–302.
- [26] P. MATTILA, *Hausdorff dimension, orthogonal projections and intersections with planes*, Annales Academiae Scientiarum Fennicae. Series A I. Mathematica, **1**, (1975), 227–244.
- [27] P. MATTILA, *The Geometry of Sets and Measures in Euclidean Spaces*, Cambridge University Press, Cambridge, (1995).
- [28] M. MENCEUR, A. BEN MABROUK AND K. BETINA, *The multifractal formalism for measures, review and extension to mixed cases*, Anal. Theory Appl., **32**, (2016), 77–106.
- [29] J. LI, L. OLSEN AND M. WU, *Bounds for the L^q -spectra of self-similar measures without any separation conditions*, J. Math. Anal. Appl., **387**, (2012), 77–89.
- [30] L. OLSEN, *A multifractal formalism*, Advances in Mathematics, **116**, (1995), 82–196.
- [31] L. OLSEN, *Mixed generalized dimensions of self-similar measures*, J. Math. Anal. Appl., **306**, (2005), 516–539.
- [32] T. C. O’NEIL, *The multifractal spectra of projected measures in Euclidean spaces*, Chaos, Solitons and Fractals, **11**, (2000), 901–921.
- [33] J. PEYRIÈRE, *A vectorial multifractal formalism*, Proc. Sympos. Pure Math., **72**, (2004), 217–230.
- [34] B. SELMI, *Multifractal dimensions for projections of measures*, Bol. Soc. Paran. Mat., **40**, (2022), 1–15.
- [35] B. SELMI, *A note on the effect of projections on both measures and the generalization of q -dimension capacity*, Probl. Anal. Issues Anal., **5**, (2016), 38–51.
- [36] B. SELMI, *Measure of relative multifractal exact dimensions*, Advances and Applications in Mathematical Sciences, **17**, (2018), 629–643.
- [37] B. SELMI, *On the strong regularity with the multifractal measures in a probability space*, Anal. Math. Phys., **9**, (2019), 1525–1534.
- [38] B. SELMI, *On the projections of the multifractal packing dimension for $q > 1$* , Ann. Mat. Pura Appl., **199**, (2020), 1519–1532.
- [39] B. SELMI, *On the effect of projections on the Billingsley dimensions*, Asian-Eur. J. Math., **13**, (2020), 2050128.
- [40] B. SELMI, *Projection estimates for mutual multifractal dimensions*, J. Pure Appl. Math. Adv. Appl., **22**, (2020), 71–89.
- [41] B. SELMI, *Projections of measures with small supports*, Annales Universitatis Paedagogicae Cracoviensis Studia Mathematica, **20**, (2021), 5–15.
- [42] B. SELMI, *The relative multifractal analysis, review and examples*, Acta Scientiarum Mathematicarum, **86**, (2020), 635–666.
- [43] B. SELMI, *The relative multifractal densities: a review and application*, Journal of Interdisciplinary Mathematics, **24**, (2021), 1627–1644.
- [44] B. SELMI AND N. YU. SVETOVA, *On the projections of mutual $L^{q,1}$ -spectrum*, Probl. Anal. Issues Anal., **6** (2017), 94–108.
- [45] B. SELMI AND N. YU. SVETOVA, *Projections and Slices of measures*, Commun. Korean Math. Soc., **36**, (2021), 327–342.
- [46] N. YU. SVETOVA, *Mutual multifractal spectra I: Exact spectra*, Tr. Petrozavodsk. Gos. Univ. Ser. Mat., **11**, (2004), 41–46.
- [47] N. YU. SVETOVA, *Mutual multifractal spectra II: Legendre and Hentschel-Procaccia spectra, and spectra defined for partitions*, Tr. Petrozavodsk. Gos. Univ. Ser. Mat., **11**, (2004), 47–56.

- [48] N. YU. SVETOVA, *The property of convexity of mutual multifractal dimension*, Tr. Petrozavodsk. Gos. Univ. Ser. Mat., **17**, (2010), 15–24.