

## NECESSARY AND SUFFICIENT CONDITIONS FOR THE CONVERGENCE OF POSITIVE SERIES

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*Abstract.* We provide new necessary and sufficient conditions for the convergence of positive series developing Bertran–De Morgan and Cauchy type tests given in [M. Martin, *Bull. Amer. Math. Soc.* 47(1941), 452–457] and [L. Bourchtein et al, *Int. J. Math. Anal.* 6(2012), 1847–1869]. The obtained result enables us to extend the known conditions for recurrence and transience of birth-and-death processes given in [V. M. Abramov, *Amer. Math. Monthly* 127(2020) 444–448].

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