

ON THE GENERALISATION OF HENSTOCK–KURZWEIL FOURIER TRANSFORM

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Abstract. In this paper, a generalised integral called the Laplace integral is defined on unbounded intervals, and some of its properties, including necessary and sufficient condition for differentiating under the integral sign, are discussed. It is also shown that this integral is more general than the Henstock–Kurzweil integral. Finally, the Fourier transform is defined using the Laplace integral, and its well-known properties are established.

Mathematics subject classification (2020): 42A38, 26A39.

Keywords and phrases: Fourier transform, Henstock–Kurzweil integral, Denjoy integral, Laplace integral, Laplace derivative.

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