

EXTENDED GENERALIZED VOIGT—TYPE FUNCTIONS AND RELATED BOUNDS

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Abstract. The principal aim of this paper is to introduce extended generalized Voigt–type function which contains the classical Voigt functions $K(x,y)$ and $L(x,y)$ as their particular cases. Functional bounding inequalities, monotonicity properties, log–convexity properties and Turán–type inequality results are presented for the investigated extended generalized Voigt–type function $\Omega_{\mu,\alpha,\beta,\nu}^{\alpha',\beta'}(x,y,z)$.

Mathematics subject classification (2020): 26A51, 26D05, 26D15, 33C20, 33C65, 33E05, 33E20, 40C19, 44A20.

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