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SHARP COEFFICIENT BOUNDS OF ANALYTIC FUNCTIONS SUBORDINATE TO SHELL–LIKE CURVES CONNECTED WITH k–FIBONACCI NUMBERS

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Abstract. Let us consider the function

$$\tilde{p}_k(z) = \frac{1+\tau_k^2 z^2}{1-k\tau_k z-\tau_k^2 z^2} := 1+\sum_{n=1}^{\infty} \tilde{p}_{k,n} z^n.$$

The coefficients of the function $\tilde{p}_k(z)$ are connected with k-Fibonacci numbers:

$$\tilde{p}_{k,n} = \left(F_{k,n-1}+F_{k,n+1}\right)\tau_k^n \qquad (n=1,2,\ldots).$$

If the function p of the form $p(z)=1+p_1 z+p_2 z^2+\cdots$ satisfies

$$p(z) \prec \tilde{p}_k(z),$$

then we have

$$\begin{aligned} |p_1| &\leqslant k|\tau_k| = \left|\tilde{p}_{k,1}\right|, \\ |p_2| &\leqslant \left(k^2+2\right)\tau_k^2 = \left|\tilde{p}_{k,2}\right|, \\ |p_3| &\leqslant \left(k^3+3k\right)|\tau_k|^3 = \left|\tilde{p}_{k,3}\right|. \end{aligned}$$

In this paper, we prove that

$$|p_n| \leqslant \left(F_{k,n-1}+F_{k,n+1}\right)|\tau_k|^n = \left|\tilde{p}_{k,n}\right| \qquad (n=1,2,\ldots).$$

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