

ON MULTIPLE  $q$ -LAGUERRE POLYNOMIALSP. NJIONOU SADJANG, J. C. MÚNLÚEM MOUNCHAROU  
AND SALIFOU MBOUTNGAM\*

**Abstract.** We study  $q$ -Laguerre multiple orthogonal polynomials. These polynomials are orthogonal with respect to  $q$ -analogues of Laguerre weight functions. We focus our attention on their structural properties. Raising and lowering operators as well as Rodrigues-type formulas are obtained and their explicit representations are given. A high-order linear  $q$ -difference equation with polynomial coefficients is deduced. Moreover, we obtain the nearest neighbor recurrence relation using a  $q$ -analogue of the theorem 23.1.11 by M.E.H. Ismail in [12].

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