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LANDAU'S THEOREM FOR BIHARMONIC MAPPINGS IN SIMPLY CONNECTED DOMAINS

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Abstract. Let $\mathscr{B}_{\mathscr{BH},\Omega}(\alpha)$ symbolize the family of biharmonic α-Bloch mappings defined on a simply connected domain $\Omega \subset \mathbb{C}$ with a parameter $\alpha > 0$. In this work, we establish a Landau-type theorem for this class by extending the results for harmonic α-Bloch mappings to the biharmonic framework. Our analysis is carried out on the shifted disk

$$\Omega_\gamma := \left\{ \zeta \in \mathbb{C} : \left| \zeta + \frac{\gamma}{1-\gamma} \right| < \frac{1}{1-\gamma} \right\}, \qquad 0 \leqslant \gamma < 1,$$

which serves as a natural generalization of the unit disk. The estimates derived here provide explicit univalence radii and image bounds for biharmonic α-Bloch mappings on Ω_γ.

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