

MULTIDIMENSIONAL HARDY TYPE INEQUALITIES FOR $p < 0$ AND $0 < p < 1$

JAMES A. OGUNTUASE^{a,c}, CHRISTOPHER A. OKPOTI^{b,c},
LARS-ERIK PERSSON^c AND FRANCIS K. A. ALLOTEY^d

Abstract. In this paper we establish some new multidimensional Hardy type inequalities for the cases $p < 0$ and $0 < p < 1$. These inequalities complement, generalize and unify most of the existing results of this type in the literature e.g. those in [4] and [9]. Some of the results are new also for the one dimensional case.

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