

FIXED POINTS AND GENERALIZED HYERS–ULAM STABILITY OF QUADRATIC FUNCTIONAL EQUATIONS

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Abstract. Let X, Y be complex vector spaces. It is shown that if a mapping $f : X \rightarrow Y$ satisfies

$$f(x + iy) + f(x - iy) = 2f(x) - 2f(y) \quad (0.1)$$

or

$$f(x + iy) - f(ix + y) = 2f(x) - 2f(y) \quad (0.2)$$

for all $x, y \in X$, then the mapping $f : X \rightarrow Y$ satisfies

$$f(x + y) + f(x - y) = 2f(x) + 2f(y)$$

for all $x, y \in X$.

Furthermore, we prove the generalized Hyers-Ulam stability of the functional equations (0.1) and (0.2) in complex Banach spaces.

Mathematics subject classification (2000): 39B72, 47H10.

Key words and phrases: Quadratic mapping, fixed point, quadratic functional equation, generalized Hyers-Ulam stability.

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