

ON VAN DE LUNE – ALZER’S INEQUALITY

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Abstract. In this paper it is shown that the inequality known in the literature as Alzer’s inequality (1993), has already been known since 1975, and is due to Jan van de Lune. A review of different methods in proving Van de Lune - Alzer’s inequality and generalizations in a several directions, is given. It is shown how some results and proofs can be corrected, refined and extended. New results, inspired by the generalization of Van de Lune - Alzer’s inequality for increasing convex sequences presented by N. Elezović and J. Pečarić, are obtained.

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