

ON A BOUNDED SUBCLASS OF CERTAIN ANALYTIC FUNCTIONS SATISFYING A DIFFERENTIAL INEQUALITY

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Abstract. In the present paper, using Jack's lemma, the authors investigate the differential inequality

$$\left| \left(1 - \alpha \right) \frac{I_p(n, \lambda) f(z)}{z^p} + \alpha \frac{I_p(n+1, \lambda) f(z)}{I_p(n, \lambda) f(z)} - 1 \right| < \mu, \quad z \in E$$

regarding multivalent functions defined by multiplier transformation in the open unit disk $E = \{z : |z| < 1\}$. As consequences, sufficient conditions for univalence, starlikeness and strongly starlikeness of certain analytic functions are obtained.

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