

ON YOUNG'S INEQUALITY AND ITS REVERSE FOR BOUNDING THE LORENZ CURVE AND GINI MEAN

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Abstract. The performance of the Young integral inequality is investigated for bounding the Lorenz curve and the Gini index. The study relies on a comparison of reverse Young type integral inequalities. The resulting approximation and bounds for the Lorenz curve and the Gini index are compared with previous results.

Mathematics subject classification (2000): 62H20, 62G10, 62P20, 26D15.

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